EPFL

L3 – Velocity Triangles

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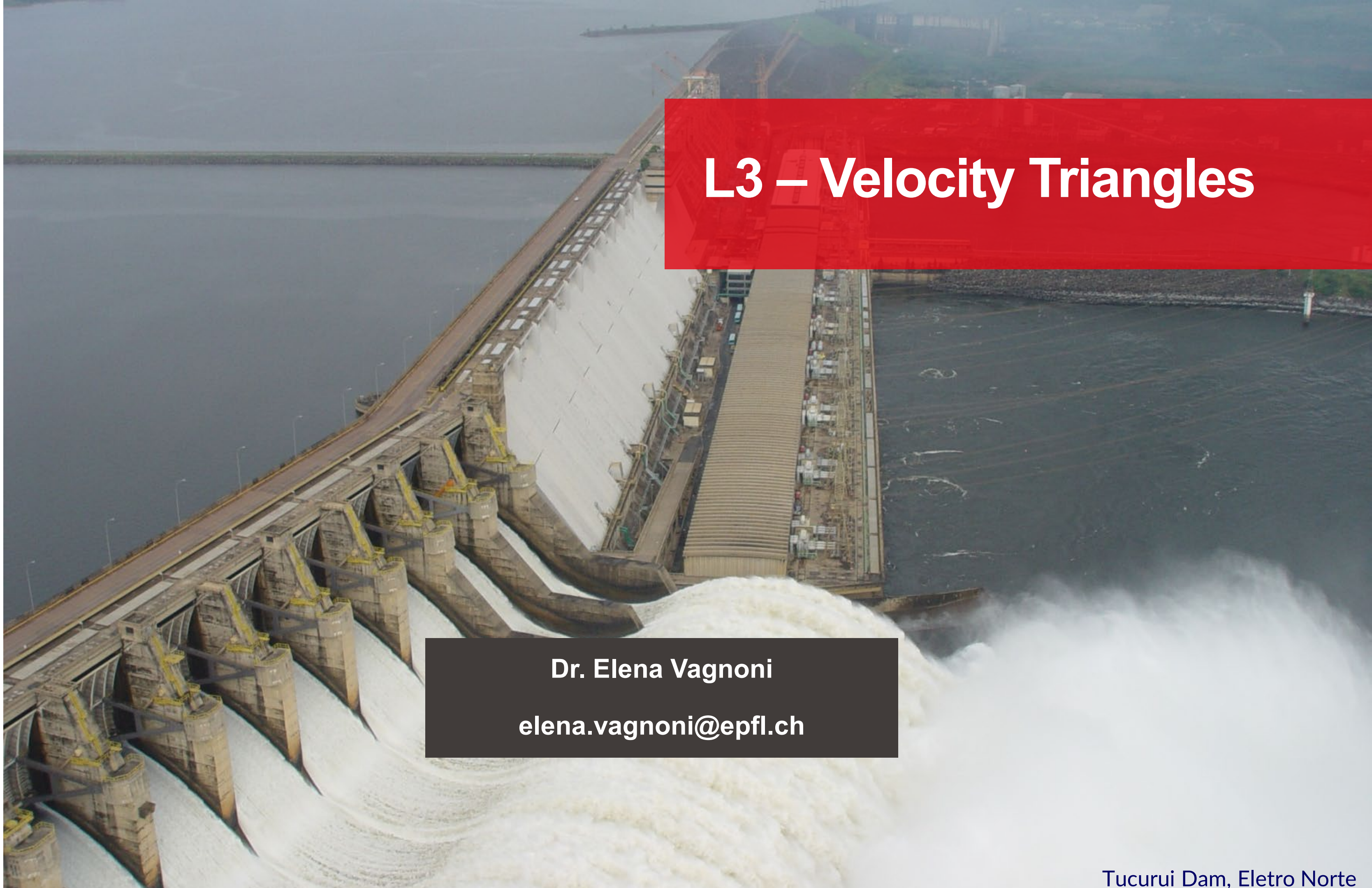

Tucurui Dam, Eletro Norte

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Topics of the lecture

- Flow field
- Runner balance
- Euler equation
- Velocity Triangles
- Runner and impeller characteristic curves

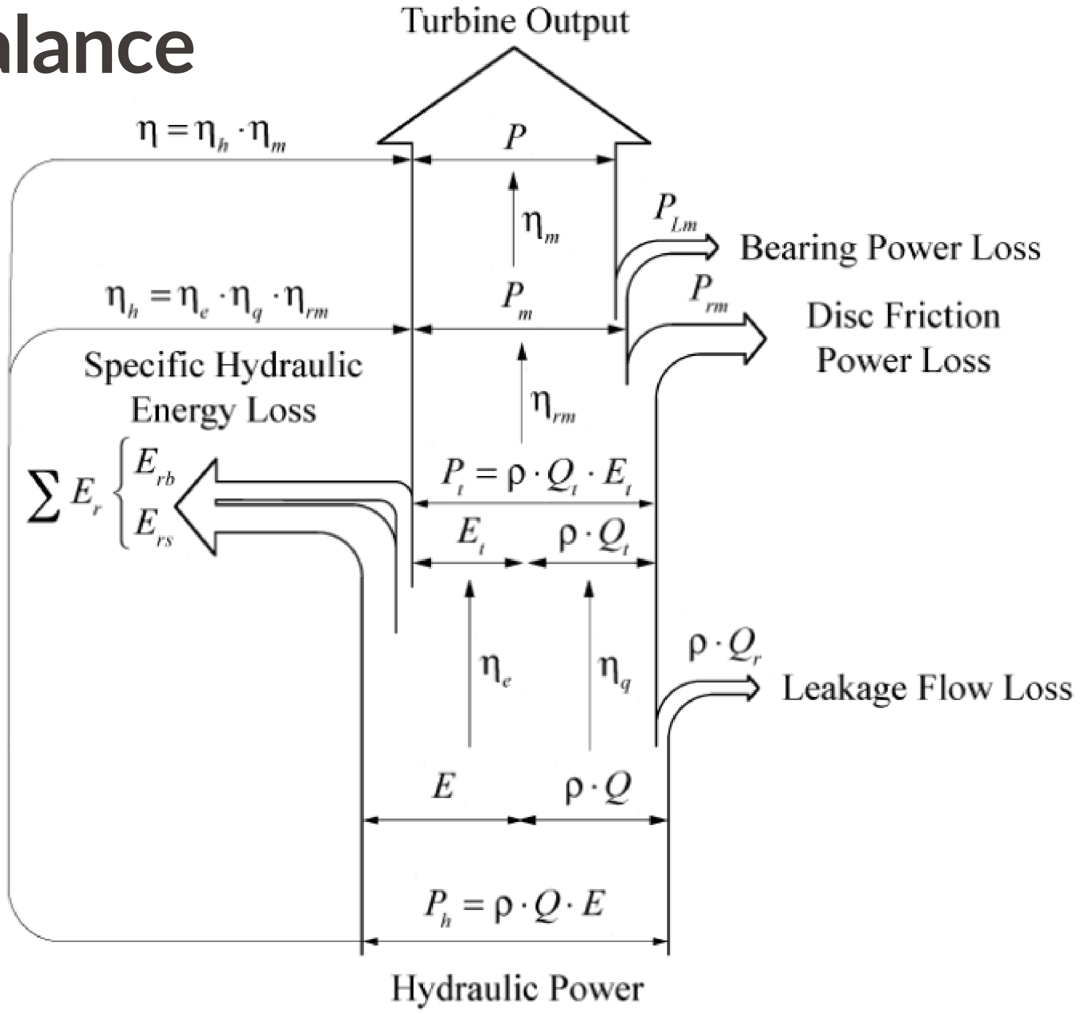
From L2: Power Balance

- Extracted Energy

$$E_t = \frac{P_t}{\rho Q_t}$$

- Driving Torque

$$P_t = \vec{\omega} \cdot \vec{T}_t$$



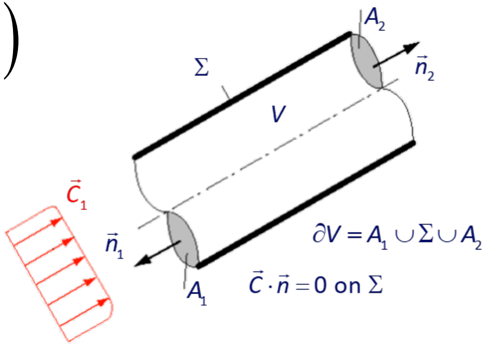
From L2: Specific Hydraulic Energy

- Local Mean Flow Specific Energy

$$h = \overbrace{\frac{p}{\rho}}^{\text{Pressure}} + \underbrace{g \times Z}_{\text{Gravity Potential}} + \overbrace{\frac{\vec{C}^2}{2}}^{\textit{Kinetic}} + Cste \quad \left(\mathrm{J \cdot kg^{-1}}\right)$$

- Discharge

$$Q = \int_A \vec{C} \cdot \vec{n} dA \quad \left(\mathrm{m^3 \cdot s^{-1}}\right)$$



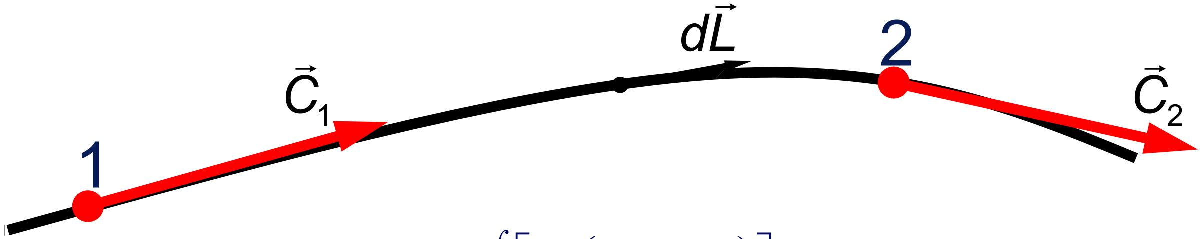
- Power

$$P_h = \int_A \rho h_t \vec{C} \cdot \vec{n} dA = \int_A \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} dA \quad (\mathrm{W})$$

From L2: Specific Hydraulic Energy

Specific Energy Balance

- Local Balance Along a Streamline



$$h_1 = h_2 - \int_{\widehat{12}} \left[\vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho} \right) \right] \cdot d\vec{L} + \int_{\widehat{12}} \frac{\partial \vec{C}}{\partial t} \cdot d\vec{L}$$

From L2: Specific Hydraulic Energy

Mean Flow and Power Balance

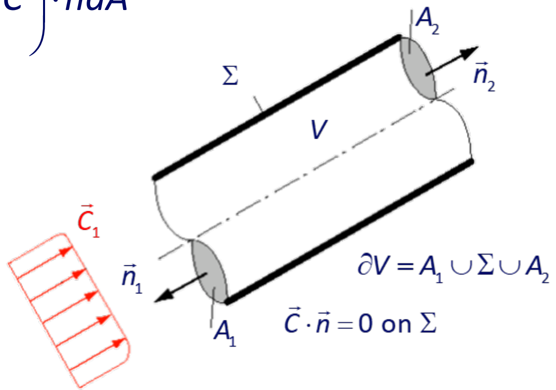
- Flow Power Balance for a Pipe Domain V

$$\int_{A_1 \cup A_2} \rho h_t \vec{C} \cdot \vec{n} dA = -\int_V \overbrace{\frac{\partial}{\partial t}\left(\frac{\vec{C}^2}{2}\right)}^{\equiv 0 \text{ if Stationary}} \rho dV + \overbrace{\int_{A_1 \cup A_2} \left(\rho\left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho}\right) \cdot \vec{C}\right) \cdot \vec{n} dA}^{\equiv 0 \text{ if Homogeneous}}$$

$$-\int_V (\Phi + \Pi) \rho dV \quad (\mathrm{W})$$

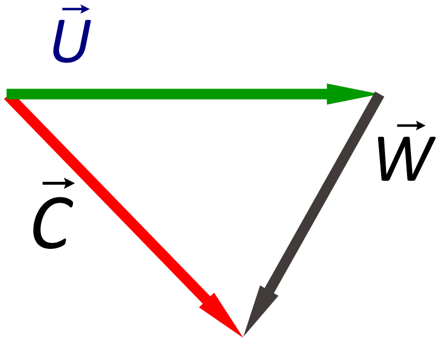
- Flow Power Dissipation

$$P_{r1\div 2} = -\int_V \left(\underbrace{\overbrace{2\nu \overline{\overline{D}}^2}^{\Phi=\text{Viscosity}} + \overbrace{\frac{\overline{\overline{\tau}}_t}{\rho} : \overline{\overline{D}}}^{\Pi=\text{Turbulence}}}_{\text{Specific Power } (\mathrm{W \cdot kg^{-1}})} \right) \rho dV$$



In an hydraulic turbine this is rotating → need to consider the relative flow!

Rotating Frame



- Absolute Flow Velocity

$$\vec{C} = \vec{U} + \vec{W}$$

- Rotating Velocity $\vec{U} = \vec{\omega} \times \vec{X}$

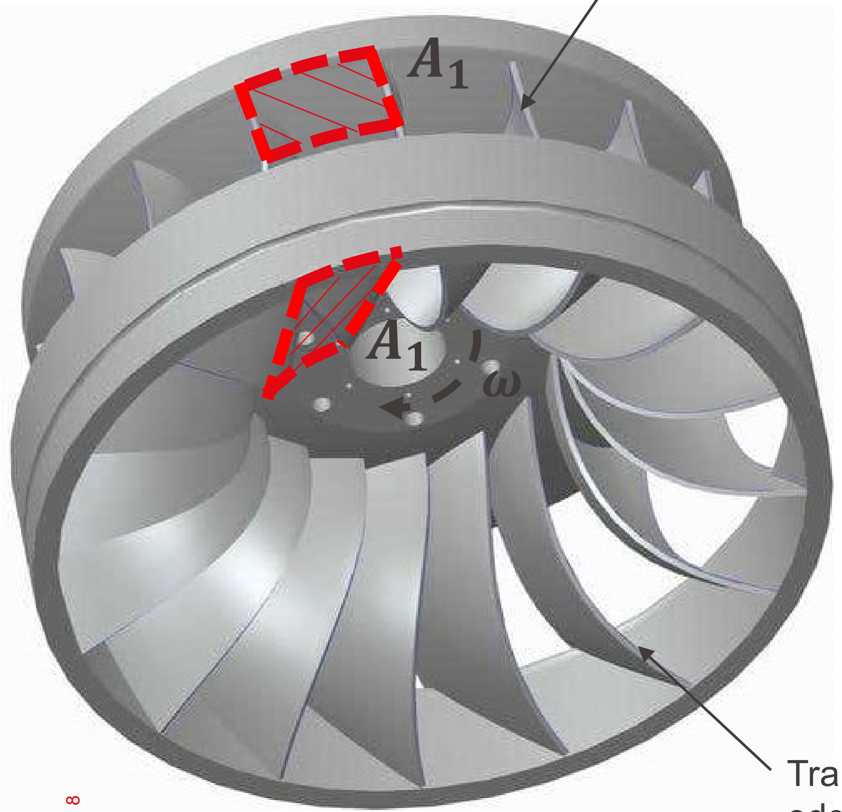
$$= \omega R$$

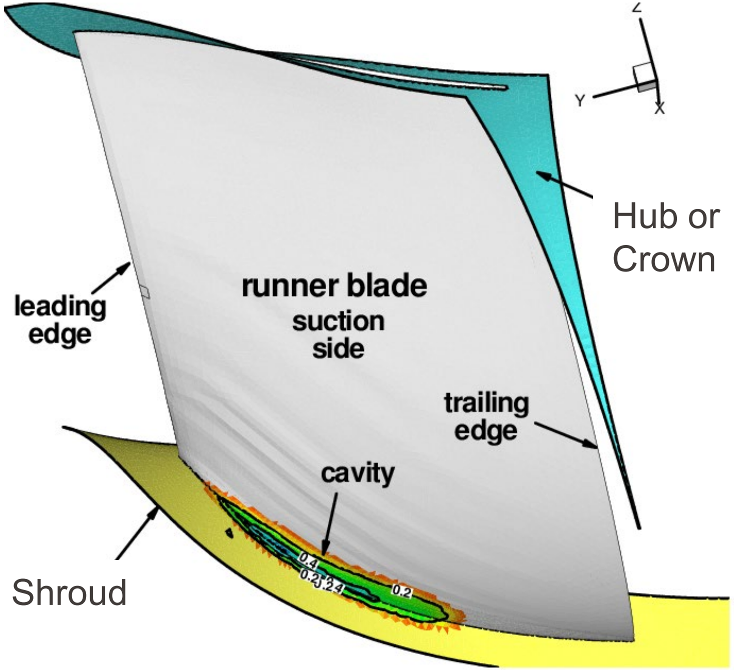
- Relative Flow Velocity

$$\vec{W} = \vec{C} - \vec{U}$$

Rotating Frame

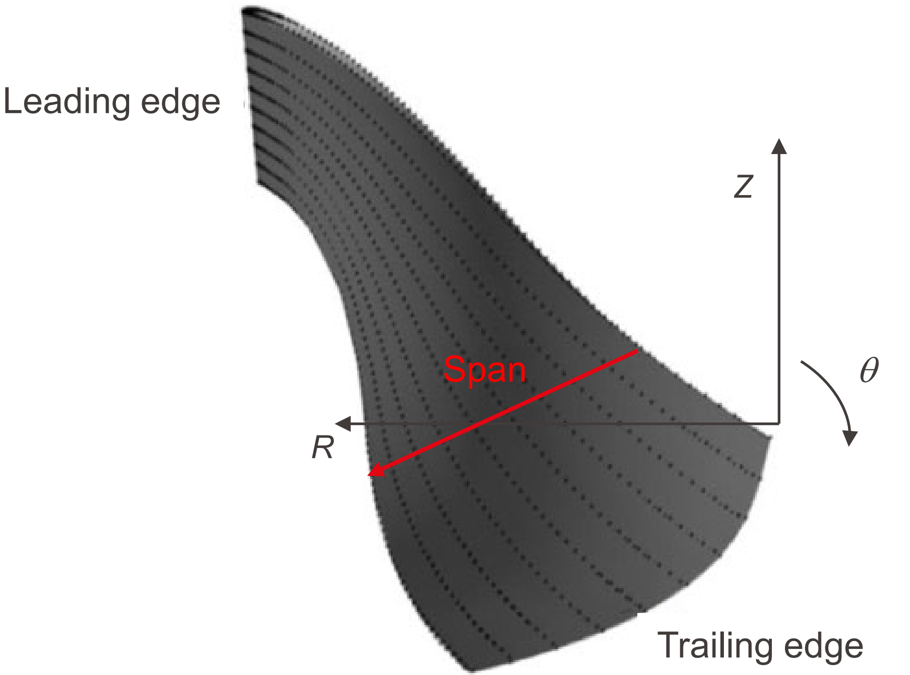
Blade shape

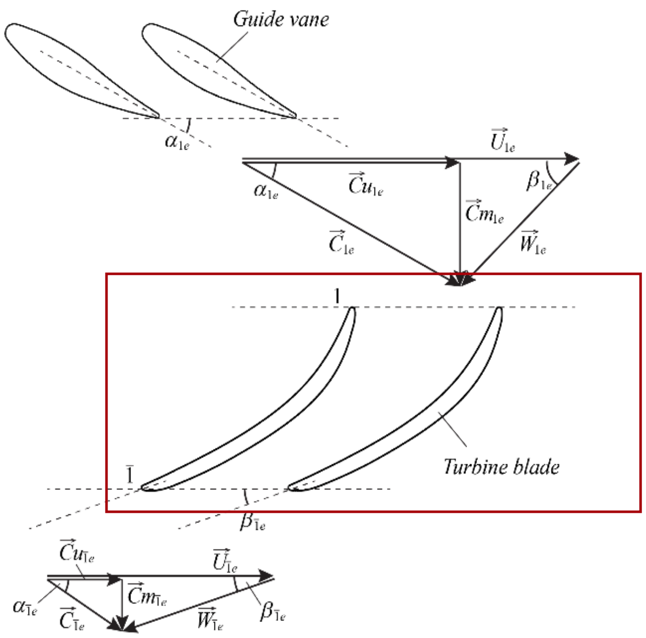




Blade to blade view

View of the blade channel projected on the 2D plane (θ,R) on a iso-span line





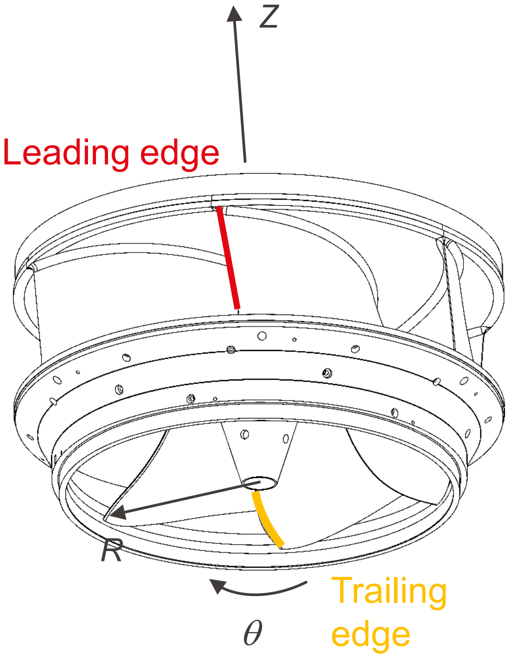
- Meridional Component $Cm = \frac{Q}{A}$
- Tangential Component $Cu = \frac{Cm}{tg\alpha} = U - \frac{Cm}{tg\beta}$

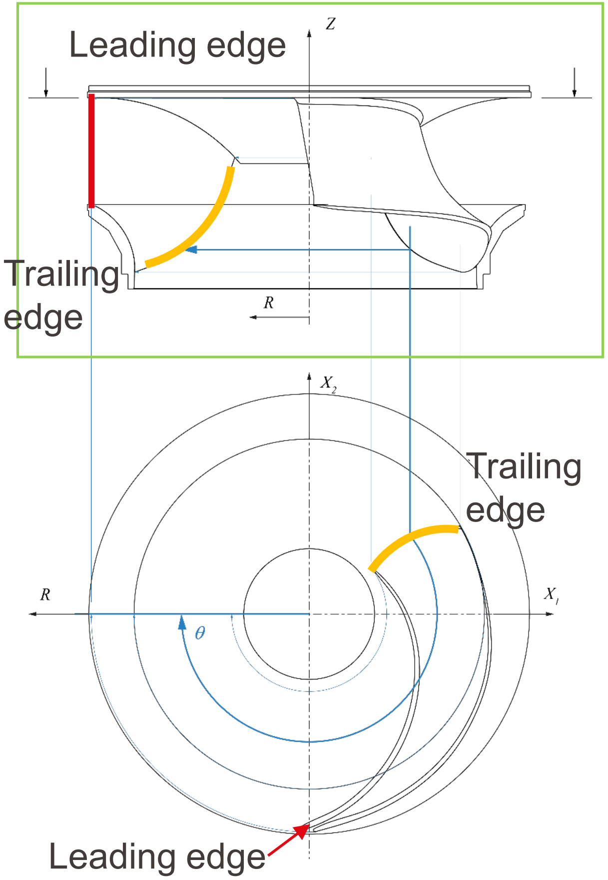
α Absolute flow angle: guide vanes

β Relative flow angle: blade angle (hypotheses)

Meridional view

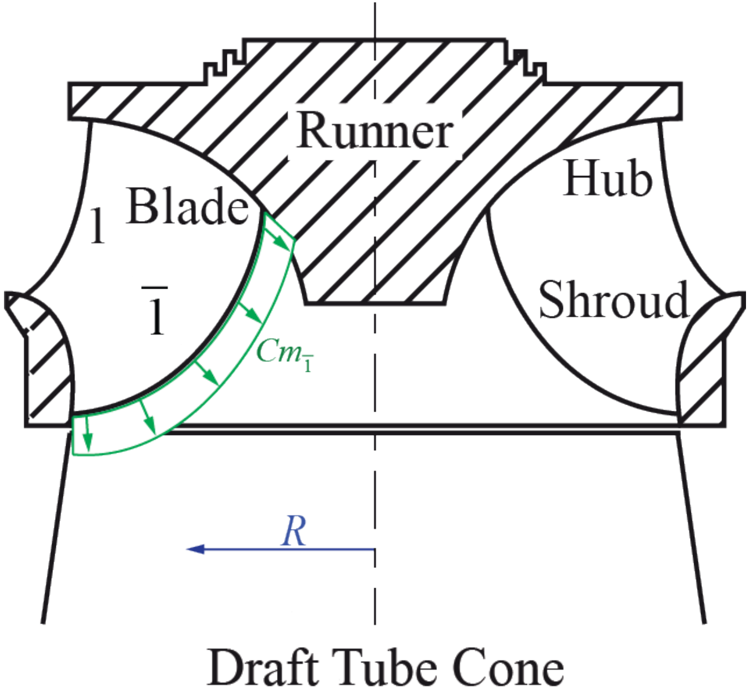
View of the blade projected on the 2D plane (Z,R)





Meridional view

Meridional View

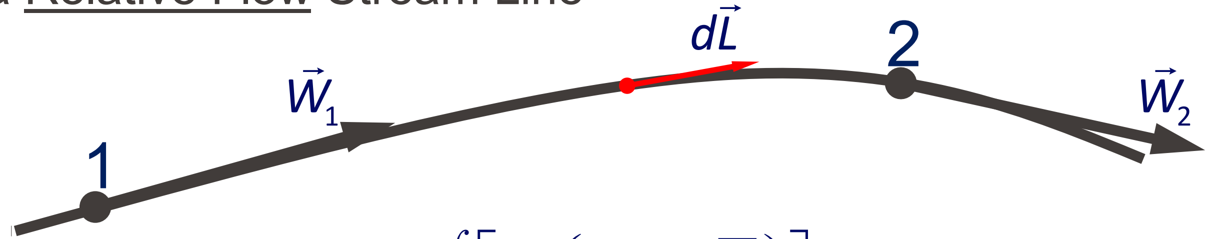


Specific Total Rothalpy

- Local rotational stagnation enthalpy: local rothalpy

$$h^{rel} = \frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} + Cste \quad \left(\mathrm{J \cdot kg^{-1}}\right)$$

- Balance Along a Relative Flow Stream Line



$$h_1^{rel} = h_2^{rel} - \int_{\overset{\frown}{12}} \left[\vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho} \right) \right] \cdot d\vec{L} + \int_{\overset{\frown}{12}} \frac{\partial \vec{W}}{\partial t} \cdot d\vec{L}$$

Relative Flow Field

- Relative Acceleration: $$\frac{D\vec{C}}{Dt} = \frac{D\vec{W}}{Dt}\bigg|_{\substack{\text{Relative}\\\text{Frame}}} \underbrace{-2\vec{\omega}\times\vec{W}}_{\text{Coriolis}} + \underbrace{\vec{\omega}\times\left(\vec{\omega}\times\vec{X}\right)}_{\text{Centrifugal}}$$

- Centrifugal Potential:

$$\vec{\omega}\times\left(\vec{\omega}\times\vec{X}\right) = -\omega^2\vec{R} = \vec{\nabla}\left(-\frac{\omega^2 R^2}{2}\right)$$

$$= \vec{\nabla}\left(-\frac{\vec{U}^2}{2}\right)$$

Relative Flow Field

- Continuity Equation $\vec{\nabla} \cdot \vec{W} = 0$
- Reynolds Averaged Navier-Stokes Equation

$$\frac{D\vec{C}}{Dt} = \frac{D\vec{W}}{Dt} - \underbrace{2\vec{\omega} \times \vec{W}}_{\text{Coriolis}} + \underbrace{\vec{\nabla}\left(-\frac{\vec{U}^2}{2}\right)}_{\text{Centrifugal}}$$

$$= -\vec{\nabla}\left(\frac{\overbrace{p}^{\text{Pressure}}}{\underbrace{\rho}_{\text{Density}}} + \overbrace{gZ}^{\text{Potential}}\right) + \vec{\nabla} \cdot \left(\underbrace{2\nu\overline{\overline{D}}}_{\text{Visc. Stress}} + \underbrace{\frac{\overline{\overline{\tau_t}}}{\rho}}_{\text{Turb. Stress}}\right)$$

$$\rightarrow \frac{D\vec{W}}{Dt} = \left[-\vec{\nabla}\left(\frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2}\right)\right] + \left(2\vec{\omega} \times \vec{W}\right) + \vec{\nabla} \cdot \left(2\nu\overline{\overline{D}} + \frac{\overline{\overline{\tau_t}}}{\rho}\right)$$

Turbine runner specific total rothalpy balance

- Rothalpy: $h^{rel} = \frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} + Cste \quad \left(\mathrm{J \cdot kg^{-1}}\right)$

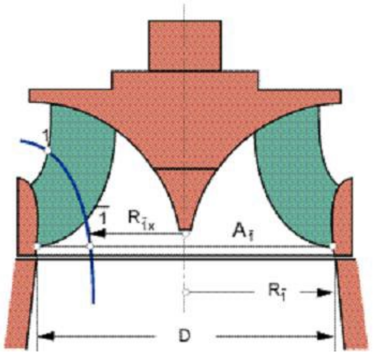
- Balance on a streamline: $h_1^{rel} = h_2^{rel} - \int_{\overset{\frown}{12}} \left[\vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho} \right) \right] \cdot d\vec{L} + \int_{\overset{\frown}{12}} \frac{\partial \vec{W}}{\partial t} \cdot d\vec{L}$

- Balance in the runner:

$$\int_{A_1 \cup A_{\bar{1}}} \left(\frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} \right) \vec{W} \cdot \vec{n} dA = \int_{A_1 \cup A_{\bar{1}}} \left[\left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}}{\rho} \right) \cdot \vec{W} \right] \cdot \vec{n} dA +$$

$$- \int_V \left(\underbrace{2\nu \overline{\overline{D}}^2}_{\Phi = \text{Viscosity}} + \underbrace{\frac{\overline{\overline{\tau}}_t}{\rho} : \overline{\overline{D}}}_{\Pi = \text{Turbulence}} \right) dV + \int_V \frac{\partial}{\partial t} \left(\frac{\vec{W}^2}{2} \right) dV$$

$$\vec{C} \cdot \vec{n} = \underbrace{\vec{U} \cdot \vec{n}}_{=0} + \vec{W} \cdot \vec{n} = \vec{W} \cdot \vec{n}$$

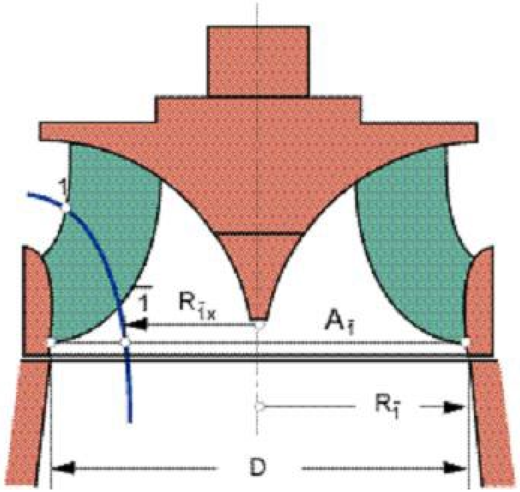


$$\partial V = A_1 \cup \Sigma_b \cup A_{\bar{1}}$$

Turbine runner hydraulic power balance

$$\int_{A_1}\left(\frac{p}{\rho}+gZ+\frac{\vec{C}^2}{2}\right)\rho\vec{C}_1\cdot\vec{n}_1 dA=\int_{A_{\bar{1}}}\left(\frac{p}{\rho}+gZ+\frac{\vec{C}^2}{2}\right)\rho\vec{C}_{\bar{1}}\cdot\vec{n}_{\bar{1}} dA+P_t+P_{rb}+$$

$$-\int_{A_1\cup A_{\bar{1}}}\rho\left[\left(2\nu\,\overline{\overline{D}}+\frac{\overline{\overline{\tau}}_t}{\rho}\right)\cdot\vec{C}\right]\cdot\vec{n}dA+\int_V\frac{\partial}{\partial t}\left(\frac{C^2}{2}\right)\rho dV$$



$$\partial V = A_1\cup\Sigma_b\cup A_{\bar{1}}$$

The change of hydraulic power of the mean flow through the runner-impeller is due to:

1. the power transfer between the flow and the runner ;
2. the rate of energy internal dissipation by both viscosity and turbulent production;
3. the net power budget of both viscous and turbulent stresses;
4. the rate of the mean flow kinetic energy change.

Turbine runner balance

- Rothalpy: $$\int_{A_1\cup A_{\bar 1}}\left(\frac{p}{\rho}+gZ\right)\vec{C}\cdot\vec{n}dA=-\int_{A_1\cup A_{\bar 1}}\left(-\frac{\vec{U}^2}{2}+\frac{\vec{W}^2}{2}\right)\vec{C}\cdot\vec{n}dA+\int_{A_1\cup A_{\bar 1}}\left[\left(2\nu\,\overline{\overline{D}}+\frac{\overline{\overline{\tau}}}{\rho}\right)\cdot\vec{C}\right]\cdot\vec{n}dA-\int_V\left(\overbrace{2\nu\,\overline{\overline{D}}^2}^{\Phi=\text{Viscosity}}+\overbrace{\frac{\overline{\overline{\tau}}_t}{\rho}:\overline{\overline{D}}}^{\Pi=\text{Turbulence}}\right)dV+\int_V\frac{\partial}{\partial t}\left(\frac{\vec{W}^2}{2}\right)dV$$
- Power: $$P_t+P_{rb}=-\int_{A_1\cup A_{\bar 1}}\left(\frac{p}{\rho}+gZ+\frac{\vec{C}^2}{2}\right)\rho\vec{C}\cdot\vec{n}dA+\int_{A_1\cup A_{\bar 1}}\rho\left[\left(2\nu\,\overline{\overline{D}}+\frac{\overline{\overline{\tau}}_t}{\rho}\right)\cdot\vec{C}\right]\cdot\vec{n}dA-\int_V\frac{\partial}{\partial t}\left(\frac{C^2}{2}\right)\rho dV$$

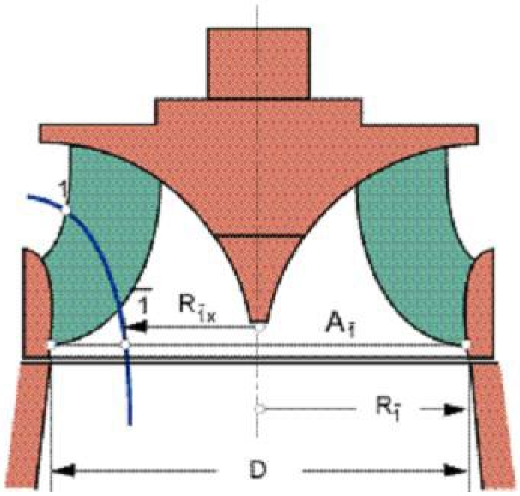
→

$$P_t=\int_{A_1\cup A_{\bar 1}}\left(-\frac{\vec{U}^2}{2}+\frac{\vec{W}^2}{2}-\frac{\vec{C}^2}{2}\right)\rho\vec{C}\cdot\vec{n}dA$$

$$P_{rb}=\int_V\left(\overbrace{2\nu\,\overline{\overline{D}}^2}^{\Phi=\text{Viscosity}}+\overbrace{\frac{\overline{\overline{\tau}}_t}{\rho}:\overline{\overline{D}}}^{\Pi=\text{Turbulence}}\right)dV$$

Hypothesis:
- Fluxes of Reynolds and viscous stresses on the inlet and outlet section are neglected
- Steady flow

Energy Conversion in the Runner
Balance along a Relative Stream Line



- Energy Balance (e.g. Turbine)

$$gH_1 = gH_{\bar{1}} + E_t + E_{rb}$$

- Local Specific Energy Balance

$$E_t = \left[gH\right]_{\bar{1}}^{1} - E_{rb}$$

$$= \left[\frac{p}{\rho} + gZ\right]_{\bar{1}}^{1} + \left[\frac{\vec{C}^2}{2}\right]_{\bar{1}}^{1} - E_{rb}$$

Energy Conversion in the Runner
Balance along a Relative Stream Line

- Extracted (or transferred) Specific Energy

$$E_t = \left[\frac{\vec{C}^2}{2} + \frac{\vec{U}^2}{2} - \frac{\vec{W}^2}{2} \right]_{\bar{1}}^{1} = \left[\frac{\vec{C}^2}{2} + \frac{\vec{U}^2}{2} - \frac{\left(\vec{C} - \vec{U}\right)^2}{2} \right]_{\bar{1}}^{1} = \vec{C}_1 \cdot \vec{U}_1 - \vec{C}_{\bar{1}} \cdot \vec{U}_{\bar{1}} = U_1 Cu_1 - U_{\bar{1}} Cu_{\bar{1}}$$

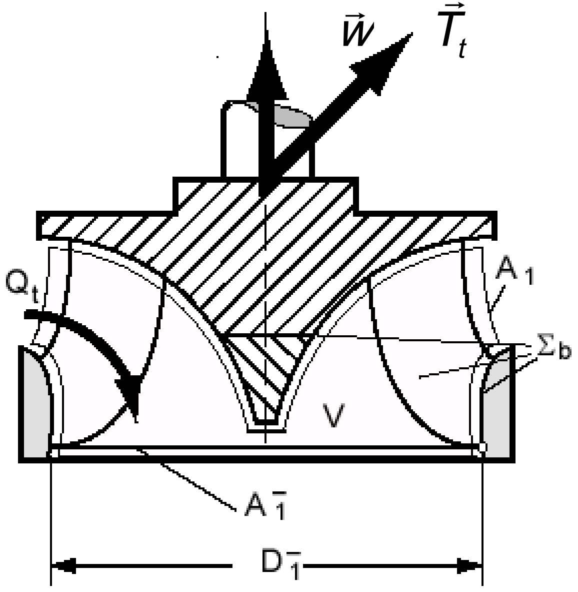
- Extracted power

$$P_t = \int_{A_1 \cup A_{\bar{1}}} \left(-\frac{\vec{U}^2}{2} + \frac{\vec{W}^2}{2} - \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} dA = - \int_{A_1 \cup A_{\bar{1}}} \left(\vec{C} \cdot \vec{U} \right) \rho \vec{C} \cdot \vec{n} dA$$

- Resulting torque

$$T_t = \frac{\rho Q_t E_t}{\omega} = \rho Q_t \frac{U_1 Cu_1 - U_{\bar{1}} Cu_{\bar{1}}}{\omega} = \rho Q_t \left(R_1 Cu_1 - R_{\bar{1}} Cu_{\bar{1}} \right)$$

Power Conversion in the Runner



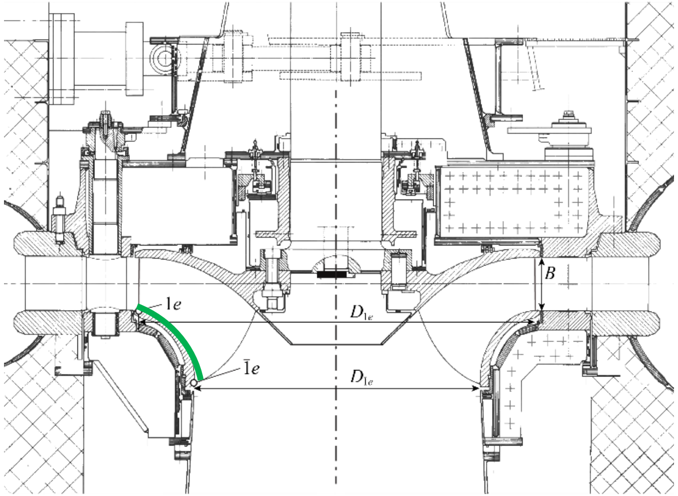
$$\partial V = A_1 \cup \Sigma_b \cup A_{\bar{1}}$$

- Extracted Power

$$P_t = \vec{\omega} \cdot \vec{T}_t$$

$$= \underbrace{- \int_{A_1 \cup A_{\bar{1}}} \left(\rho \cdot \vec{C} \cdot \vec{U} \right) \vec{C} \cdot \vec{n} dA}_{\text{Change of Angular Momentum Power (Mean Flow)}}$$

The Global Euler Equation



- 1D Equation: local form of the transferred energy by considering any particular streamline, for example, the transferred specific energy is defined by using the outer, external, streamline between the 2 points:

$$E_t = k_{C_{u1e}} \vec{C}_{1e} \cdot \vec{U}_{1e} - k_{C_{u\bar{1}e}} \vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e}$$

  - As the Euler equation is defined for the mean flow, the local form uses the following flow velocity distribution coefficients to take into account the influence of the spatial velocity distribution :

$$k_{C_{ux}} = \left| \frac{\int_{A_x} \left(\vec{C} \cdot \vec{U}\right) \vec{C} \cdot \vec{n}\, dA}{Q_t \left(\vec{C}_x \cdot \vec{U}_x\right)} \right|$$

  - Turbine Flow: Uniform Inlet & Solid Body Rotation Outlet.

Uniform flow at turbine inlet

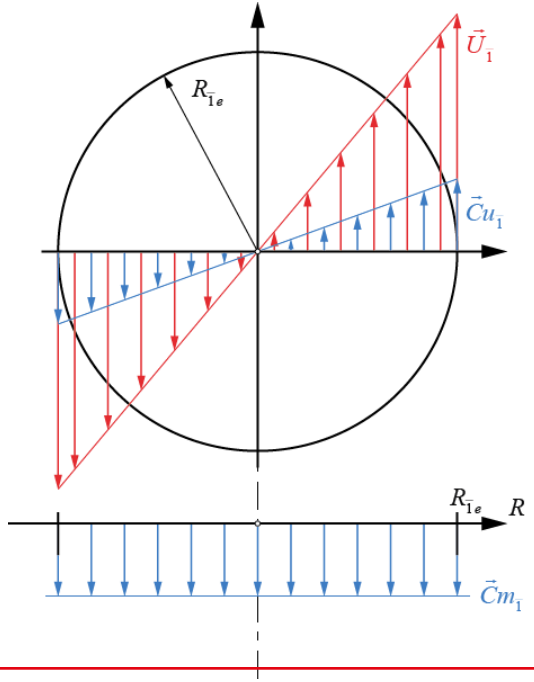
$$k_{C_{uA_{1e}}} = \left| \frac{\int_{A_{1e}} \left(\vec{C} \cdot \vec{U}\right) \vec{C} \cdot \vec{n} \, dA}{Q_t \left(\vec{C}_{1e} \cdot \vec{U}_{1e}\right)} \right| = 1$$

$$k_{C_{mA_{1e}}} = \left| \frac{\int_{A_{1e}} \vec{C} \cdot \vec{n} \, dA}{A_{1e} \, Cm_{1e}} \right| = 1$$

Solid Body Rotation at turbine outlet

$$\frac{\vec{U}}{\vec{U}_{\bar{1}e}} = \frac{R}{R_{\bar{1}e}}$$

$$\frac{C_u}{C_{u\bar{1}e}} = \frac{R}{R_{\bar{1}e}}$$



$$k_{C_{uA_{\bar{1}e}}} = \left| \frac{\int_{A_{\bar{1}e}} \left(\vec{C}\cdot\vec{U}\right) \vec{C}\cdot\vec{n}\, dA}{Q_t \left(\vec{C}_{\bar{1}e}\cdot\vec{U}_{\bar{1}e}\right)} \right|$$

$$k_{C_{mA_{\bar{1}e}}} = \left| \frac{\int_{A_{\bar{1}e}} \vec{C}\cdot\vec{n}\, dA}{A_{\bar{1}e}\, Cm_{\bar{1}e}} \right| = 1$$

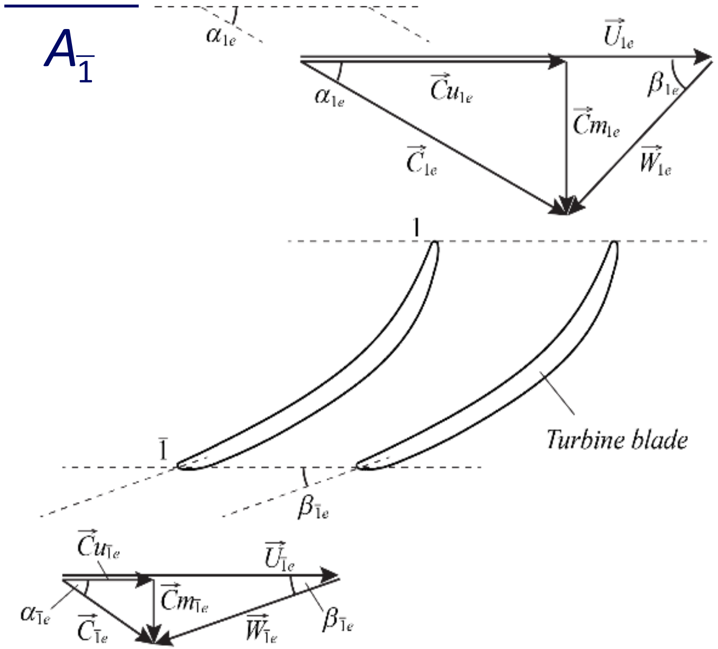
$$E_t = (1)\times\vec{C}_{1e}\cdot\vec{U}_{1e} - \left(\frac{1}{2}\right)\times\vec{C}_{\bar{1}e}\cdot\vec{U}_{\bar{1}e}$$

→ Depending on the velocity components the extracted energy will be different!

Turbine Runner Velocity Triangles and Characteristic Curve

$$E_t = k_{C_{u1e}} \vec{C}_{1e} \cdot \vec{U}_{1e} - k_{C_{u\bar{1}e}} \vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e} = U_{1e} Cu_{1e} - k_{C_{u\bar{1}e}} U_{\bar{1}e} Cu_{\bar{1}e}$$

$$E_t = -k_{u\bar{1}e} U^2_{\bar{1}e} + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\operatorname{tg} \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{U_{\bar{1}e} Q_t}{A_{\bar{1}}}$$



$$Cm = \frac{Q_t}{A}$$

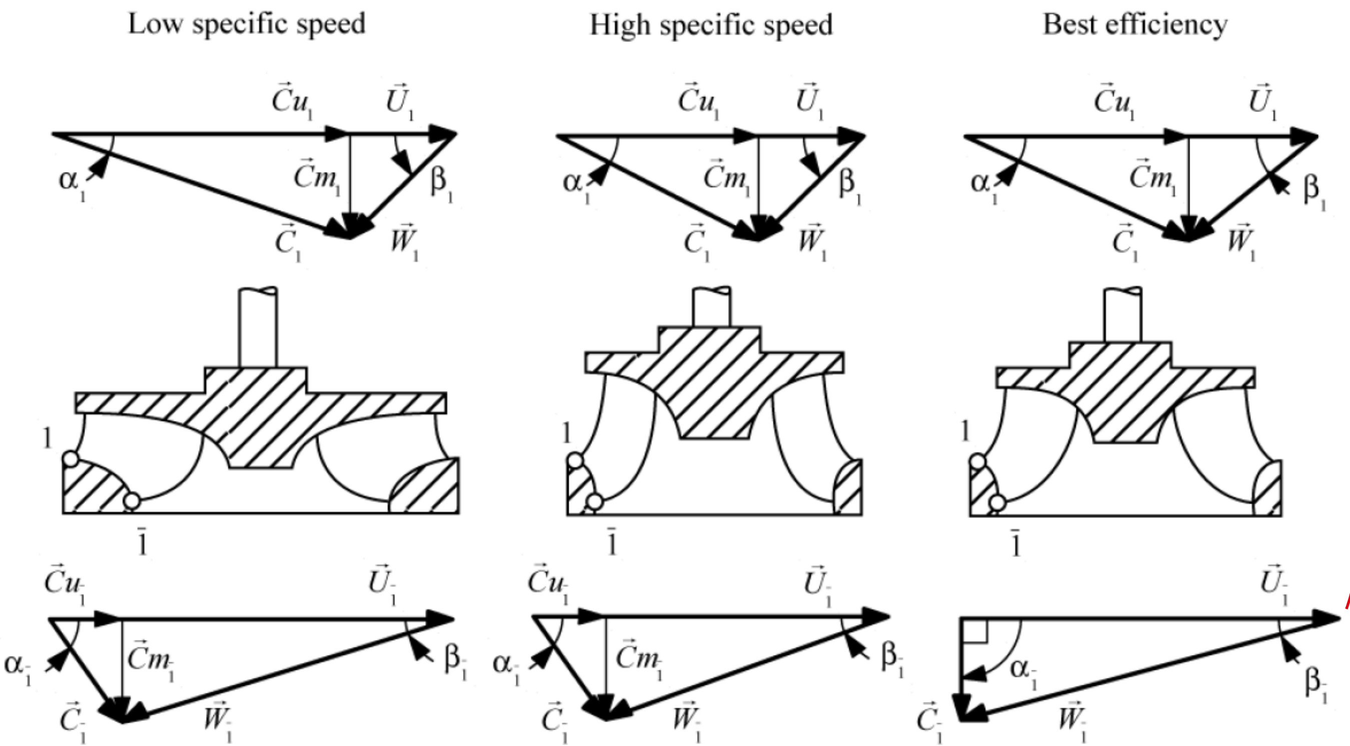
$$Cu = \frac{Cm}{tg\alpha} = U - \frac{Cm}{tg\beta}$$

Turbine Runner Velocity Triangles

$$E_t = U_{1e}Cu_{1e} - k_{C_{u\bar{1}e}} U_{\bar{1}e} Cu_{\bar{1}e}$$

Inlet

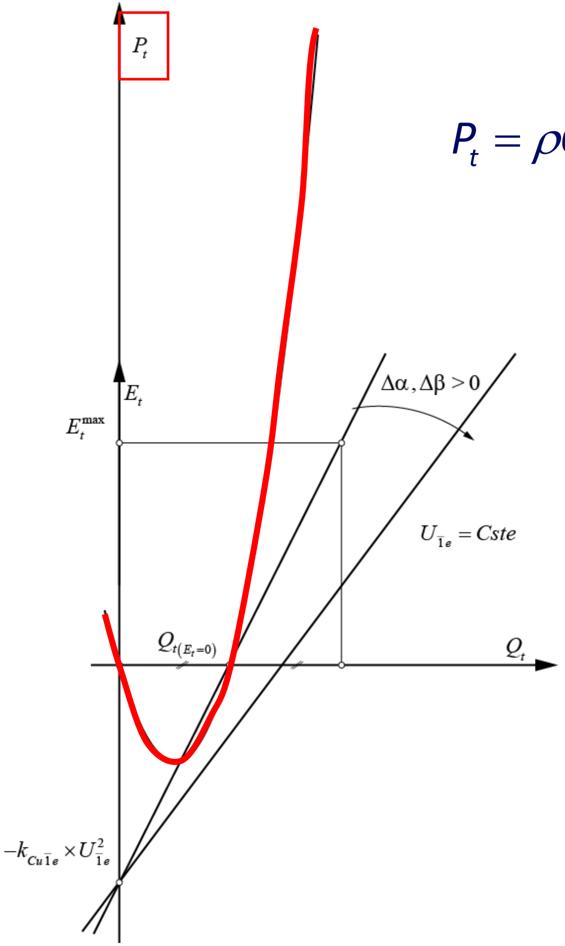
Outlet



Axial outlet flow

$$Cu_{\bar{1}e} = 0$$

Turbine Runner Characteristic Curve

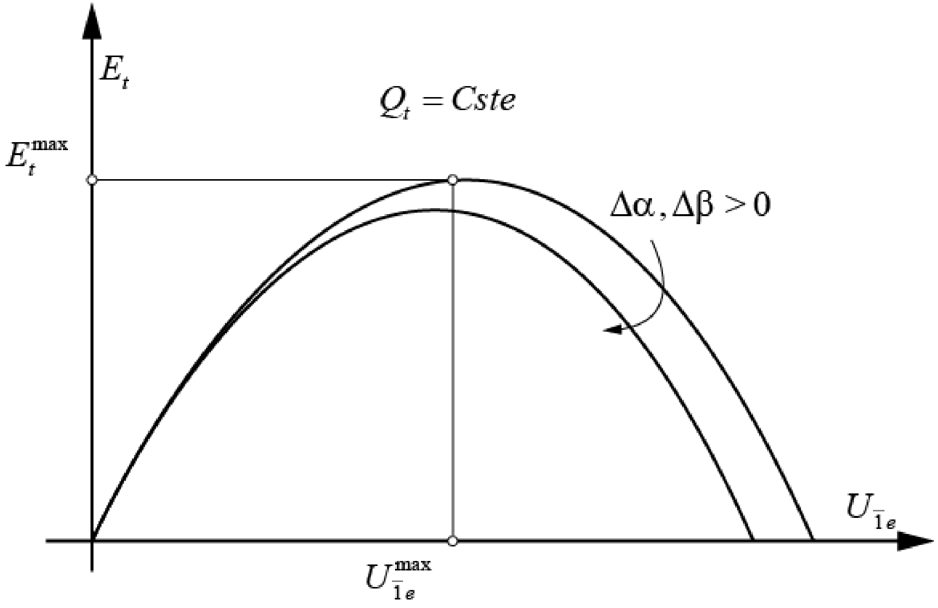


$$P_t = \rho Q_t E_t = -k_{u\bar{1}e} U^2_{\bar{1}e} \rho Q_t + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\operatorname{tg} \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{\rho U_{\bar{1}e} Q_t^2}{A_{\bar{1}}}$$

$$E_t = -k_{u\bar{1}e} U^2_{\bar{1}e} + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\operatorname{tg} \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{U_{\bar{1}e} Q_t}{A_{\bar{1}}}$$

Turbine Runner Characteristic Curve

$$E_t = -k_{u\bar{1}e} U_{\bar{1}e}^2 + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\operatorname{tg} \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{U_{\bar{1}e} Q_t}{A_{\bar{1}}}$$



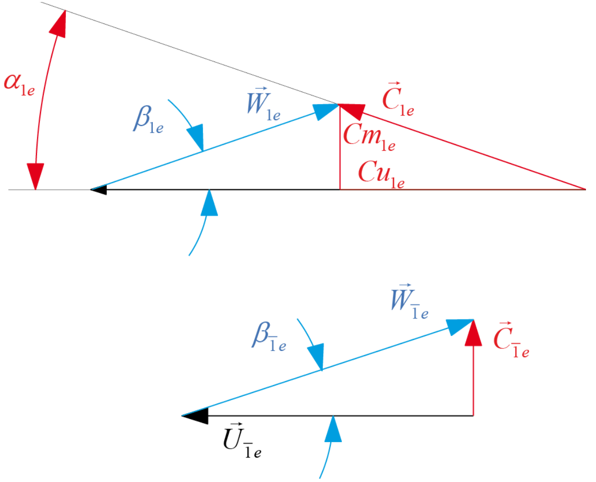
$$\frac{dE_t}{d\omega} = -2k_{u\bar{1}e} U_{\bar{1}e} + \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\operatorname{tg} \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{Q_t}{A_{\bar{1}}} = 0$$

$$U_{\bar{1}e}^{max} = \left[\frac{R_{1e} A_{\bar{1}}}{R_{\bar{1}e} A_1} \frac{1}{\operatorname{tg} \alpha_{1e}} + k_{u\bar{1}e} \frac{1}{\operatorname{tg} \beta_{\bar{1}e}} \right] \frac{Q_t}{2k_{u\bar{1}e} A_{\bar{1}}}$$

$$E_t^{max} = k_{u\bar{1}e} U_{\bar{1}e}^2$$

Pump Impeller Characteristic curve

Hypothesis: Swirl Free Inlet Flow→ axial flow at the inlet section



Outlet

Inlet

$$E_t = k_{u1e} U_{1e}^2 - \left[k_{u1e} \frac{1}{\operatorname{tg} \beta_{1e}} + \underbrace{k_{u\bar{1}e} \frac{R_{\bar{1}e}}{R_{1e}} \frac{A_1}{A_{\bar{1}e}} \frac{1}{\operatorname{tg} \alpha_{\bar{1}e}}}_{=0 \text{ If } \alpha_{\bar{1}e} = \frac{\pi}{2}} \right] U_{1e} \frac{Q_t}{A_1}$$

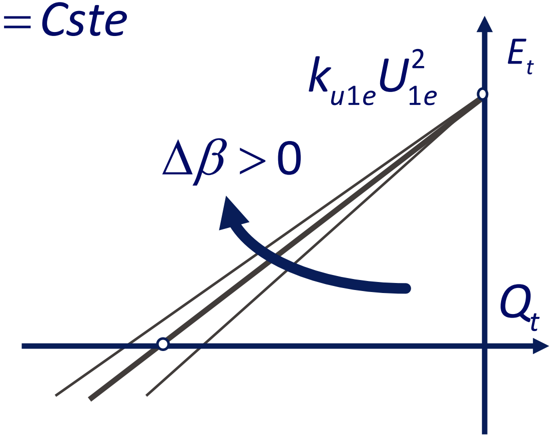
Pump Impeller Characteristic curve
Swirl Free Inlet Flow

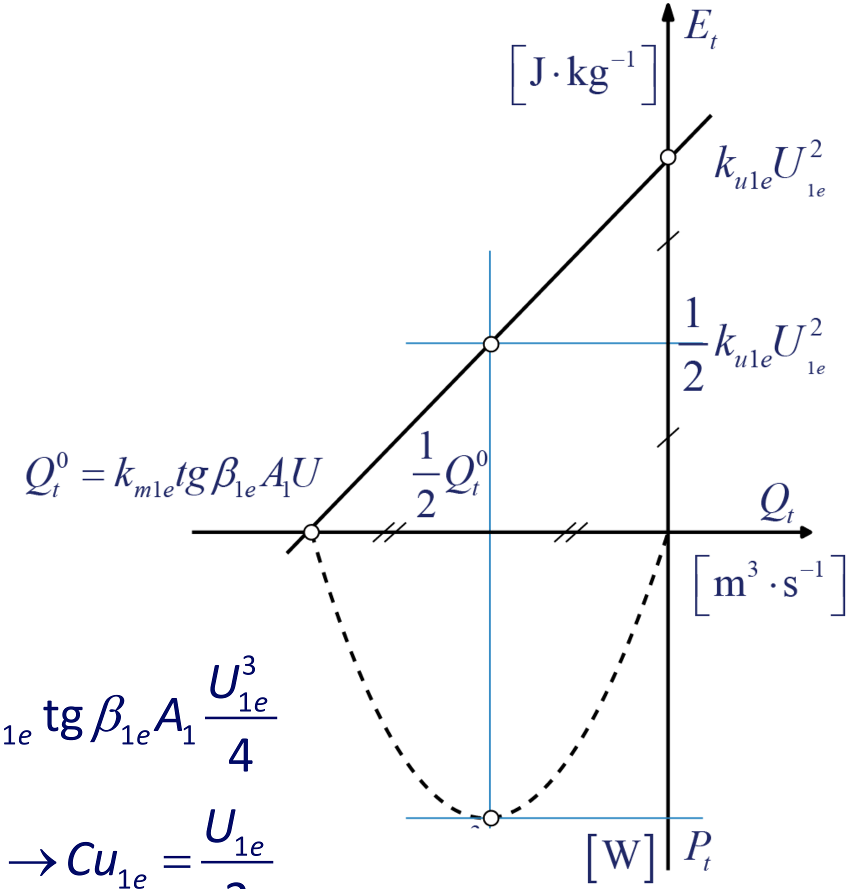
- Transferred Power $P_t = \rho Q_t U_{1e} \left[k_{u1e} U_{1e} - k_{u1e} \frac{1}{\operatorname{tg} \beta_{1e}} \frac{Q_t}{A_1} \right]$
  - No Load Conditions $P_t = 0$

$$Q_t = 0 \text{ and } Q_t^0 = tg\beta_{1e} A_1 U_{1e}$$

Pump Impeller Characteristic curve
Swirl Free Inlet Flow

$\omega = Cste$





$$P_t = P_t^{\min} = \rho k_{u1e}\, \mathrm{tg}\, \beta_{1e} A_1 \frac{U_{1e}^3}{4}$$

$$E_t^{\min} = k_{u1e}\frac{U_{1e}^2}{2} \rightarrow Cu_{1e} = \frac{U_{1e}}{2}$$

Pump Impeller Velocity Triangles
Swirl Free Inlet Flow

Outlet

Inlet